Risking Other People's Money:
Gambling, Limited Liability, and Optimal Incentives

Peter DeMarzo, Dmitry Livdan, Alexei Tchistyi

Stanford University

U.C. Berkeley

Motivation

- Financial meltdown 2008
  - Ex ante unlikely outcome
  - Ex post AIG, Lehman, Citi, Merrill Lynch, etc. suffered high losses
  - Losses were caused by divisions trading highly risky securities
  - Investors were unable to either monitor or understand actions taken by managers

- Managers enjoy limited liability and their compensation is performance based

Moral Hazard and Optimal Contracting

- Managers may seek private gain by taking on ***tail risk***
  - Earn bonuses based on short-term gains
  - Put firm at risk of rare disasters
  - Limited liability leaves them insufficiently exposed to downside risk
  - Is this the result of inefficient contracting?

- Standard contracting models
  - Focus on effort provision
  - Static and dynamic models
  - Rewards for high cash flows can be optimal
  - But does this contract lead to excessive risk-taking?

One-Period Model

- Principal/Investor(s)
  - Risk-neutral
  - Owns the company
  - Value of the company without project is *A* (large)

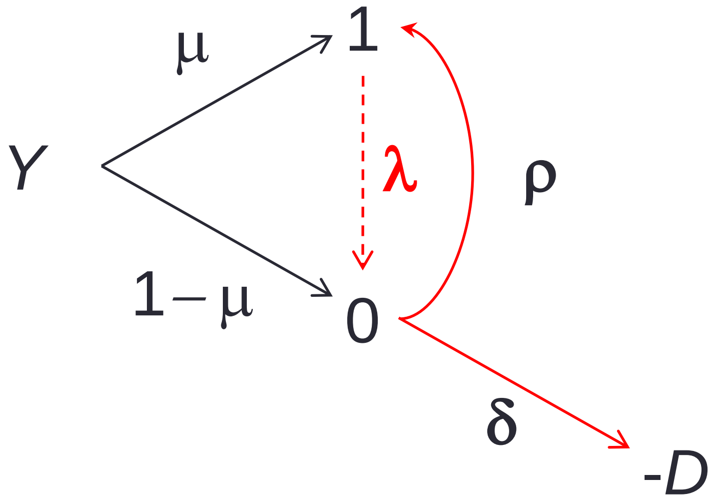
- One period risky project with payoff:

$$Y(q) = \begin{cases} 1, \text{ with probability } & \mu + q\rho \\ 0, \text{ with probability } & 1 - \mu - q(\rho + \delta). \\ \text{-}D, \text{ with probability } & q\delta \end{cases}$$

- Project risk
  - Low risk $q = 0$
  - High risk $q = 1$
  - High risk is suboptimal: $\rho - \delta D < 0$

One-Period Model

- Principal hires agent/manager to run the project
- New output *Y*, subject to two-dimensional agency problem:
  - Divert output / shirk for private benefit (λ)
  - Gamble ($\rho < \delta D$)
- How does the possibility of gambling affect contracting?



One-Period Model

- Contract specifies payoffs (w_0, w_1, w_d)
  - $w_d = 0$
  - $w_1 \geq w_0 + \lambda$
- No Gambling:
  - $\rho\,(w_1 - w_0) \leq \delta\, w_0 \quad \Leftrightarrow \quad w_0 \geq \rho\,\lambda / \delta$
  - Agent must receive sufficient rents to prevent gambling
    - Exp. payoff $= w_0 + \mu\,\lambda$

      $\geq \rho\,\lambda / \delta + \mu\,\lambda = \lambda\,(\mu + \rho / \delta) \equiv \boldsymbol{w^s}$
- Gambling:
  - Reduce agent rents: $w_0 \geq 0$
    - Exp. payoff $= w_0 + (\mu + \rho)\,\lambda \geq \lambda\,(\mu + \rho) \equiv \boldsymbol{w^g} < \boldsymbol{w^s}$
  - Suffer expected loss: $\delta\, D - \rho \equiv \Delta$

One-Period Model

- Low risk is more profitable to principal than high risk if

$$\mu - w^s \geq \mu - \Delta - w^g$$

$$\Delta \geq \lambda\,(\rho/\delta - \rho)$$

  - For small δ principal would prefer to implement high risk project or not to undertake any project

- Gambling is more costly to prevent when probability of disaster is low
  - Limited liability prohibits harsh punishment of agent for gambling,
  - Expected loss δw_0 is low when δ is low,
  - Unless agent's compensation w_0 and w^s are high

Contract Conditional on Disaster

- If we cannot punish agent for gambling it may be cheaper to reward him for not gambling ex post

- Can the agent be rewarded for not gambling ex post?
  - Oil spills
    - Absence does not mean gambling did not occur – perhaps we just got lucky?
  - Earthquakes
    - If the building survives an earthquake, that *is* evidence that the builder did not cut corners
  - Financial crisis
    - If a bank survives it while other banks fail, that *is* evidence that the bank did not gamble

Bonus for not Gambling

- No gambling: pay bonus *b* if no loss (- *D*) given disaster

$$\rho\ (w_1 - w_0) \leq \delta\ (w_0 + b)$$

- Contract without gambling that maximizes principal payoff:

$$w_d = 0,\ w_0 = 0,\ w_1 = \lambda,\ b = \lambda\ \rho\ /\ \delta.$$

  - Bonus *b* may be large, but expected bonus payment is not

$$\delta\ b = \lambda\ \rho$$

  - Exp. payoff for Agent $= \lambda\ \mu + \delta\ b \quad = \lambda\ \mu + \rho\ \lambda \equiv$ $\boldsymbol{w^g}$

- In that case, *no gambling is always optimal*

Implementation Using Put Options

- Agent is given out-of-money put options on companies that are likely to be ruined in the "disaster" state
  - Caveat: Agent can collect the payoff from the options only if his company remains in a good shape

- Potential downside of using put options
  - Creates incentives to take down competitors

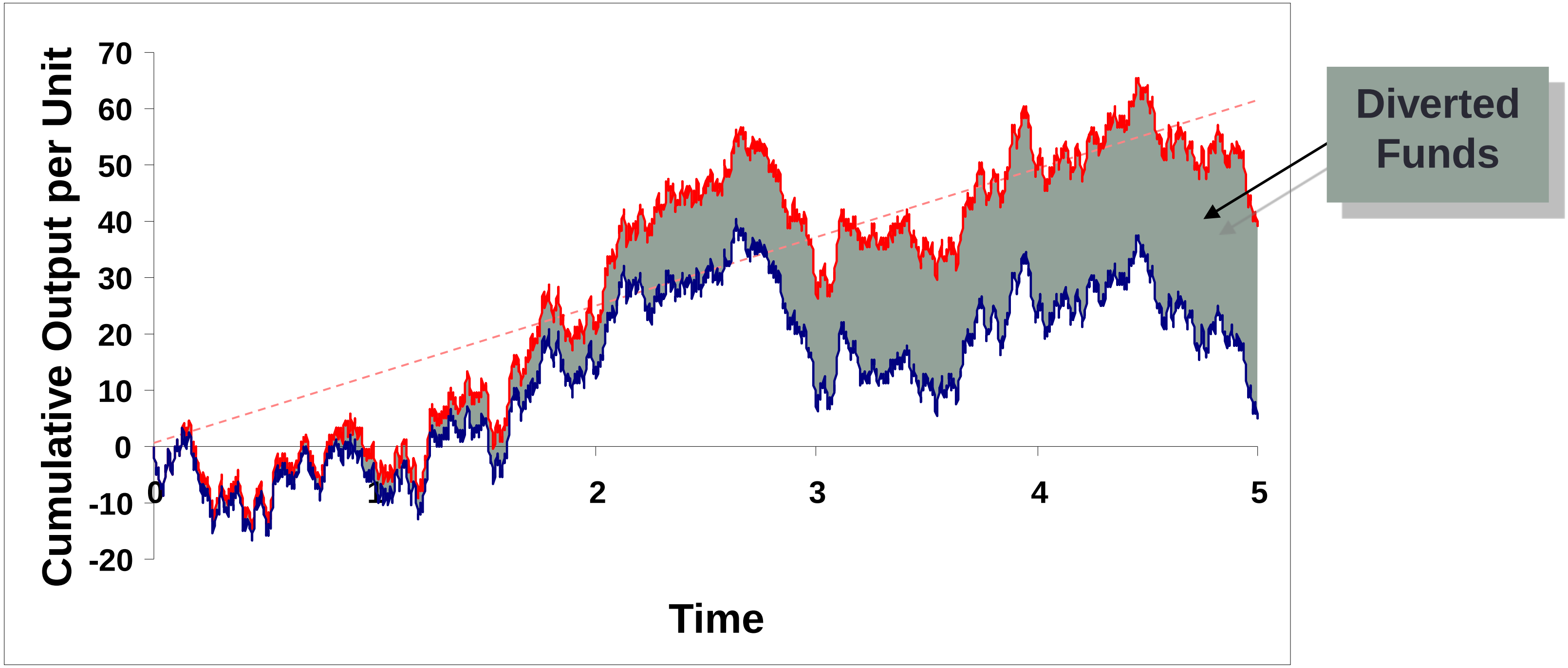
- Comprehensive cost-benefit analysis is needed

Dynamic Model

- A simple model (DS 2006)
  - Cumulative cash flow: $dY = \mu\, dt + \sigma\, dZ$
  - Agent can divert cash flows and consume fraction $\lambda \in (0, 1]$
  - Alternative interpretation: drift μ depends on agent's effort
    - Earn private benefits at rate λ per unit reduction in drift
- Gambling with tail risk
  - Gambling raises drift to $\mu + \rho$: $dY = (\mu + \rho)\, dt + \sigma\, dZ$
  - Disaster arrives at rate δ, destroying the franchise and existing assets D if the agent gambled

Basic Agency Problem

- Interpretations
  - Cash Flow Diversion
  - Costly Effort (work/shirk)



The Contracting Environment

- Agent reports cash flows
- Contract specifies, as function of the history of cash flows:
  - The agent's compensation $dC_t \geq 0$
  - Termination / Liquidation
    - Agent's outside option = 0
    - Investors receive value of firm assets, $L < \mu/r$
- Contract curve / value function:

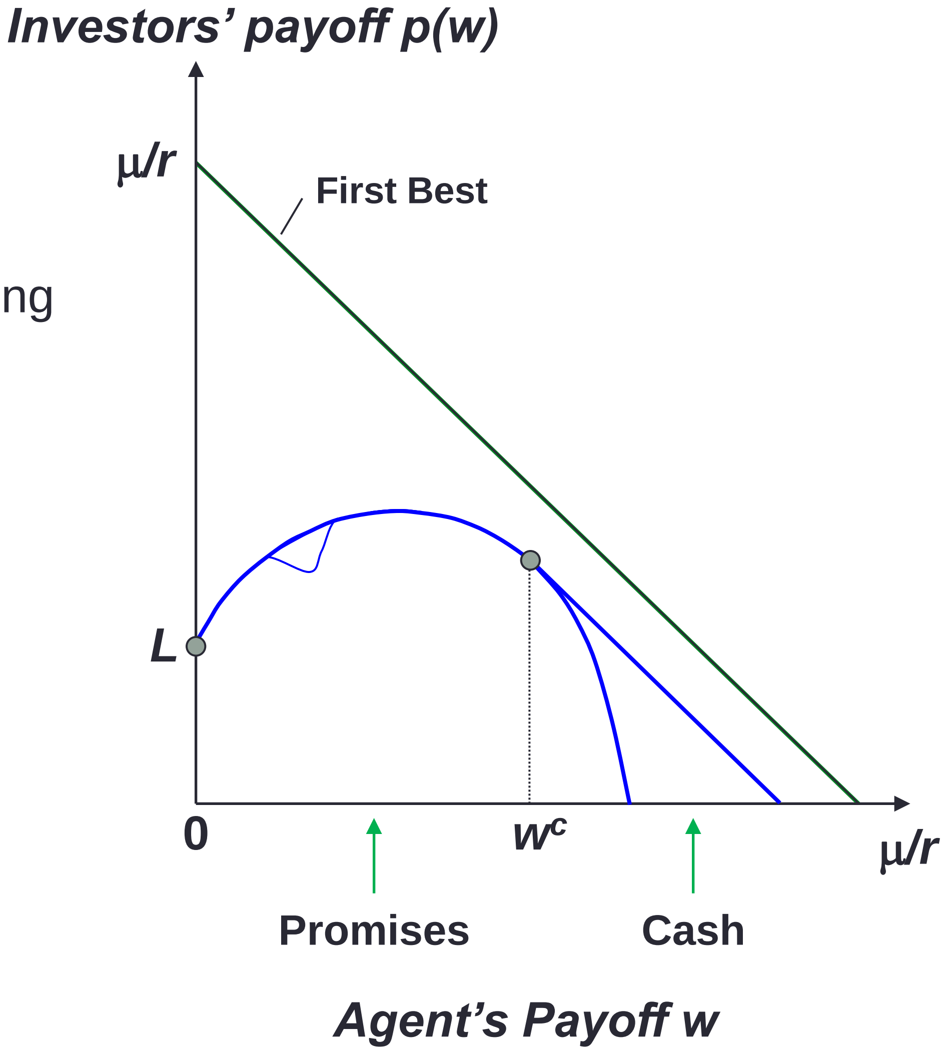
  $p(w)$ = max investor payoff given agent's payoff w

  - Provide incentives via cash dC_t or promises dw_t
  - Tradeoff: Deferring compensation eases future IC constraints, but costly given the agent's impatience

Solving the Basic Model

- First-Best Value Function
  - $p^{FB}(w) = \mu/r - w$
- Basic Properties
  - Positive payoff from stealing/shirking

    $\Rightarrow$ $p(0) = L$
  - Public randomization
    $\Rightarrow$ $p(w)$ is weakly concave
  - Liquidation is inefficient
    $\Rightarrow$ $p(w) + w \leq \mu/r$
- Cash Compensation
  - $\Rightarrow$ $p'(w) \geq -1$
  - Pay cash if $w > w^c$
  - Use promises if $w \leq w^c$



Basic Model cont'd

- Agent's Future Payoff *w*
  - Promise-keeping
    - $E[dw] = \gamma\, w\, dt$
  - Incentive Compatibility
    - $\partial w / \partial y \geq \lambda$

$$\Rightarrow \; dw = \gamma w dt + \lambda (dy - E[dy])$$

$$= \gamma w dt + \lambda \sigma \, dZ$$

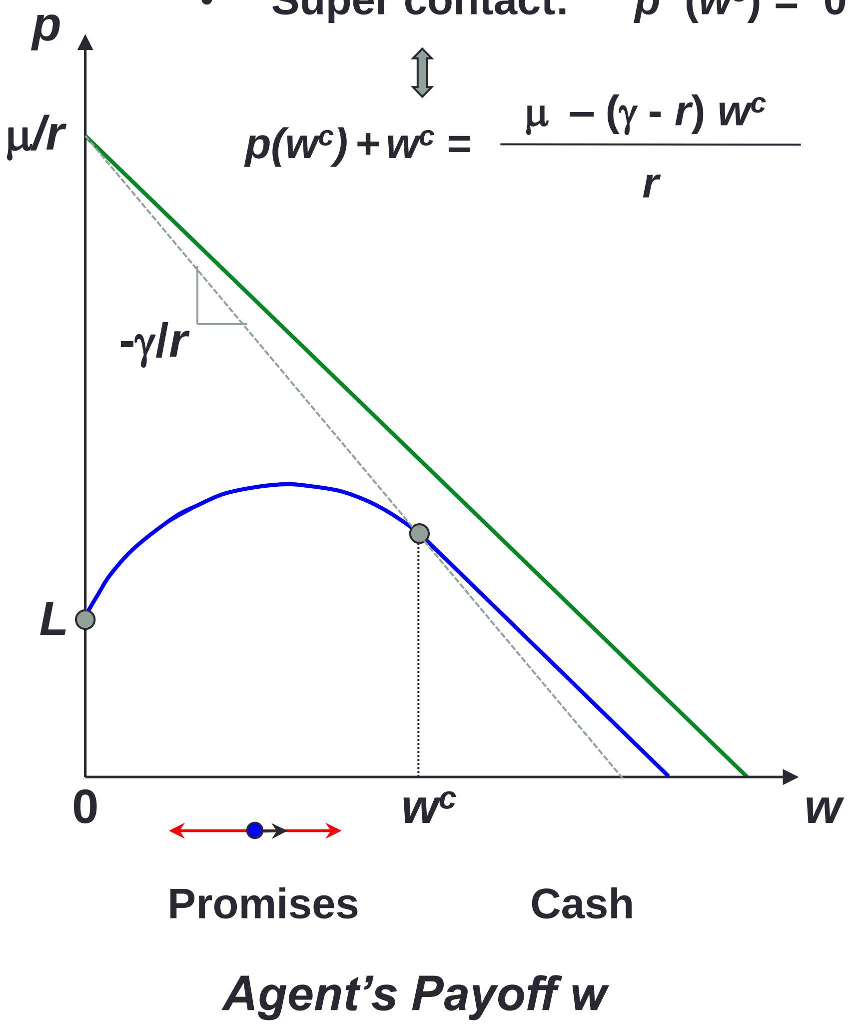
- Investor's Payoff: HJB Equation

$$\underbrace{rp}_{\text{Req. Return}} = \underbrace{\mu}_{E[FCF]} + \underbrace{\gamma\, wp' + \tfrac{1}{2}\lambda^2\sigma^2 p''}_{E[dp]}$$

Boundary Conditions:

- **Termination:** $p(0) = L$
- **Smooth pasting:** $p'(w^c) = -1$
- **Super contact:** $p''(w^c) = 0$

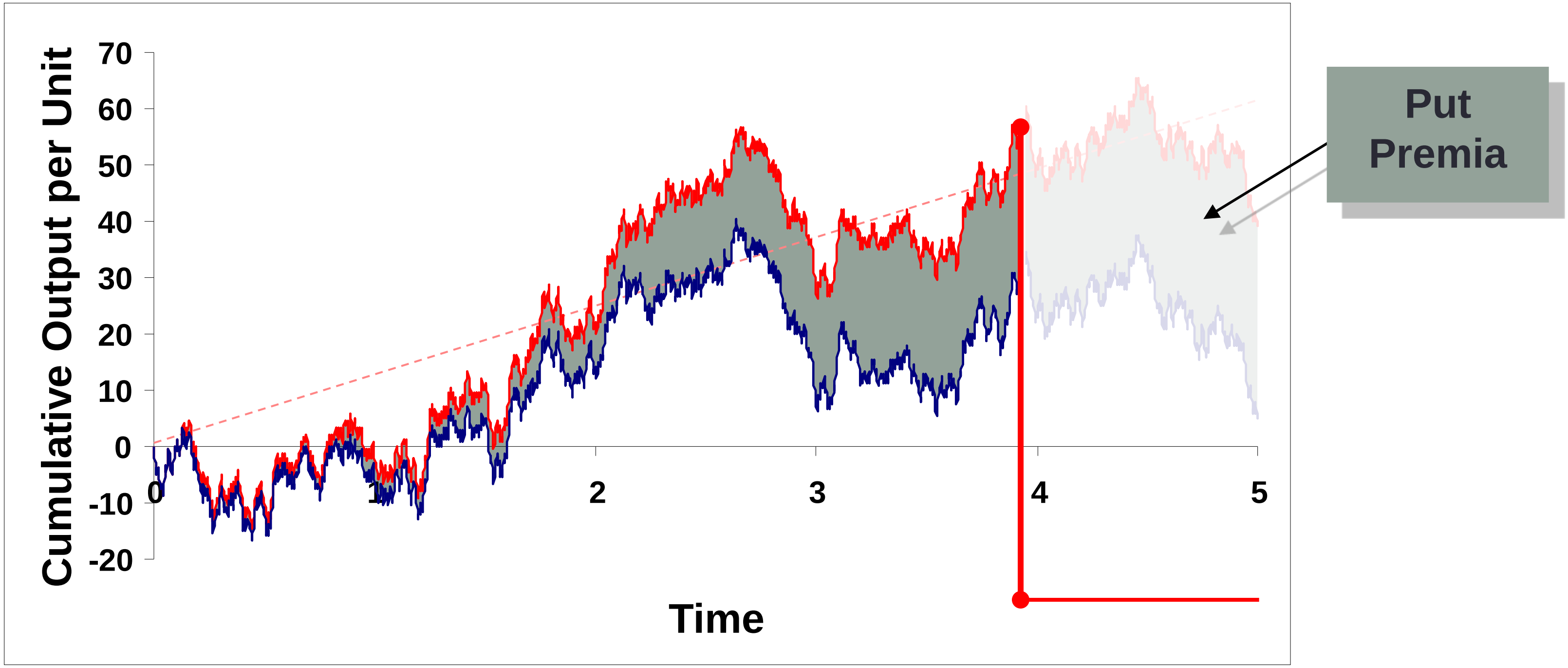
⇕



$$p(w^c) + w^c = \frac{\mu - (\gamma - r)\, w^c}{r}$$

The Gambling Problem

- Agent may increase profits by taking on tail risk
  - E.g. selling disaster insurance / CDS / deep OTM puts – earn ρ *dt*
  - Risk of disaster that wipes out franchise – arrival rate δ *dt,* loss *D*



The Gambling Problem

- Agent's incentives
  - Gain from gambling: $\lambda \rho \, dt$
  - Potential loss: w_t, with probability $\delta \, dt$
  - Agent will gamble if $\lambda \rho > \delta w_t$ or

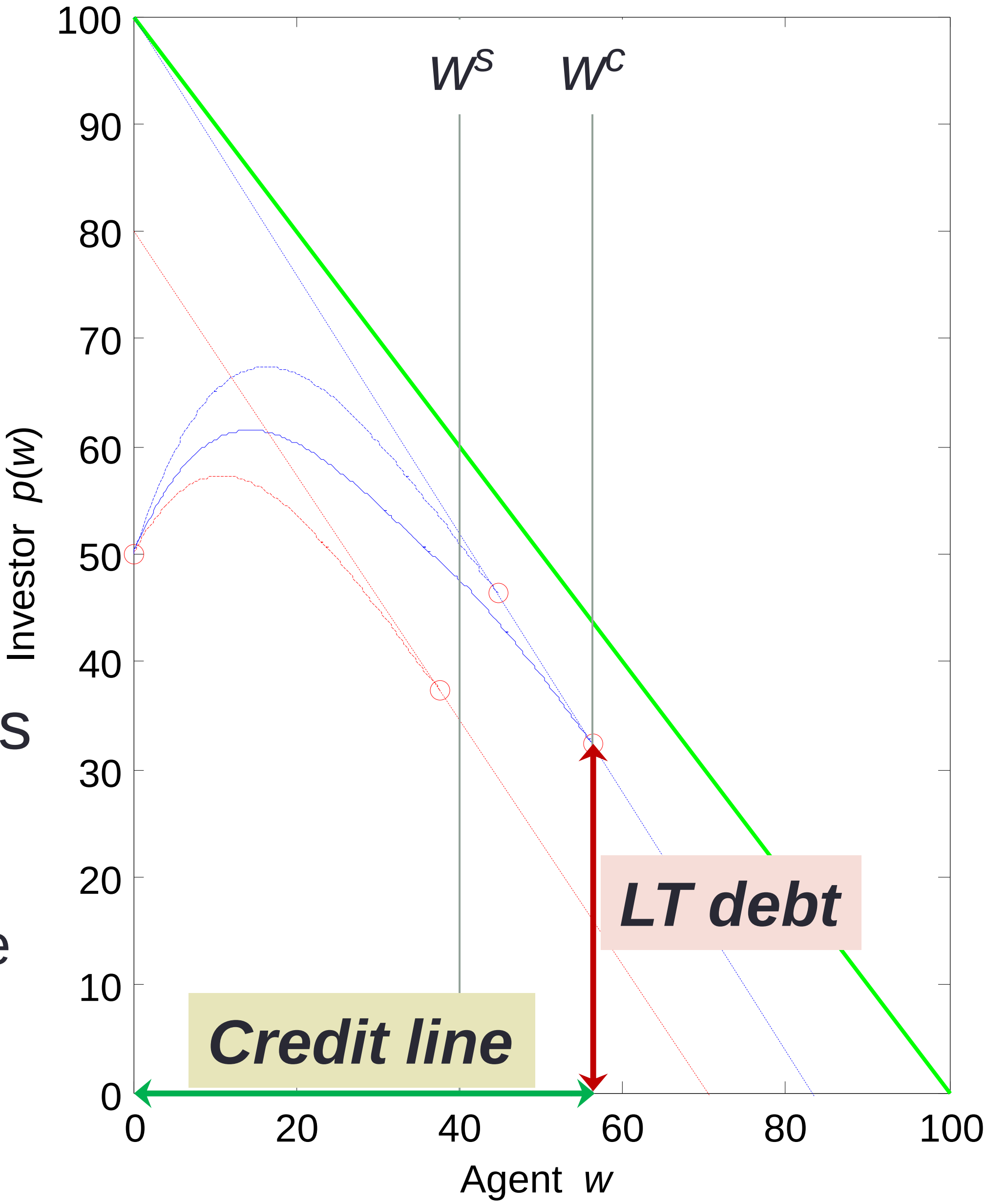
$$w_t < w^s \equiv \lambda \rho / \delta$$

  - Agent will gamble if not enough "skin in the game"

- Gambling region
  - Contract dynamics: $dw = (\gamma + \delta) w dt + \lambda (dy - E[dy])$
  - Value function: $(r + \delta) p^g = (\mu + \rho - \delta D) + (\gamma + \delta) w p^{g\prime} + ½ \lambda^2 \sigma^2 p^{g\prime\prime}$
    - Increased impatience
  - Smooth pasting: $p(w^s) = p^g(w^s)$, $p'(w^s) = p^{g\prime}(w^s)$

Example

- First Best = 100
  - $\mu = 10$, $r = 10\%$, $\gamma = 12\%$, $\sigma = 8$, $L = 50$, $\lambda = 1$
- Cash if $w > 56$
  - $w^c = 56$
- Gamble if $w < 40$
  - $\rho = 2$, $\delta = 5\%$, $w^s = 40$, $D = 0$
- Compare to pure cases
  - Longer deferral of compensation
  - Greater use of credit line vs. debt (more financial slack)



Ex-Post Detection and Bonuses

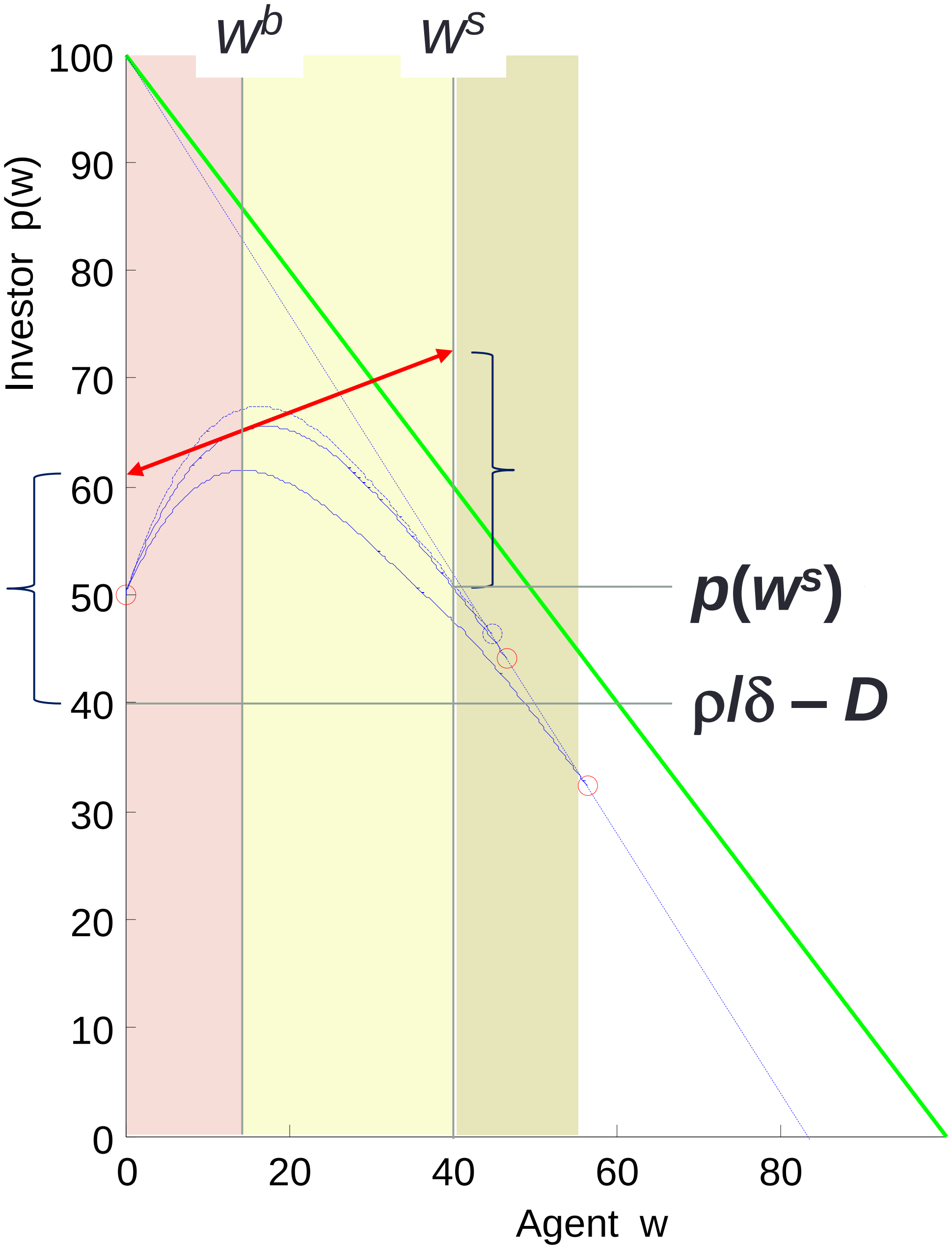
- Suppose disaster states are observable
  - Earthquakes, Financial Crises, …
  - Can we avoid gambling by offering bonuses to survivors ex-post?
- How large a bonus?
  - If $w_t \geq w^s$: no bonus is needed to provide incentives
  - If $w_t < w^s$: increase w_t to w^s if firm survives disaster : $b_t = w^s - w_t$
- Bonus region
  - Contract dynamics: $dw = [(\gamma + \delta) w - \delta w^s] dt + \lambda (dy - E[dy])$
  - Value function:

$$(r + \delta) p^b = (\mu + \delta p^b(w^s)) + [(\gamma + \delta) w - \delta w^s] p^{b\prime} + ½ \lambda^2 \sigma^2 p^{b\prime\prime}$$

  - Smooth pasting …

Optimal Bonuses

- Bonus payments:
  - substantially improve investor payoff
  - reduce need for deferred comp / financial slack / harsh penalties (no jumps)
- For low enough w_t, gambling is still optimal



Summary

- The double moral hazard problem is likely to be important in firms where risk-taking can be easily hidden
- Risk-taking is likely to take place
  - Probability of disaster is low
  - After a history of poor performance, when the agent has little “skin” left in the game
- As a result, optimal policies will have increased reliance on deferred compensation
- When the “safe” practices can be verified ex-post, we can mitigate risk-taking via bonuses
- When effort costs are convex, we should expect reductions in effort incentives as a means to limit risk-taking, with a jump to high powered incentives in the gambling region