a dynamic model for optimal covenants in loan contracts

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Covenants

- 94% of private debt agreements include at least one financial covenant (Demiroglu and James (2010))
- more that 90% of the contracts are renegotiated (Roberts and Sufi, 2009; Nikolaev, 2015)
- the average bank loan is renegotiated five times (Roberts (2015))
- More than 75% of all debt contract renegotiations modify at least one of the restrictive or financial covenants (Roberts (2015))

Renegotiations

- Renegotiations are costly
- Negative reactions of stock market can be a consequence of late and frequent renegotiations (Godlewski (2015))
- Credit constraints (and further renegotiations) has been proven to have the effect on real economy (Ersahin and Irani (2018))

Continuous Time Model

- A firm investing in a project, which is financed through a bank loan that contains covenants
- The value of the firm's assets (or investment project)

$$dX_t = \mu X_t dt + \sigma X_t dW_t$$

- Covenant: if the value of the firm assets (or the value of the investment project) falls to a specified level, then the bank can incur losses

$$\alpha(t) = e^{-r(T-t)} S_T \ \rho$$

Continuous Time Model

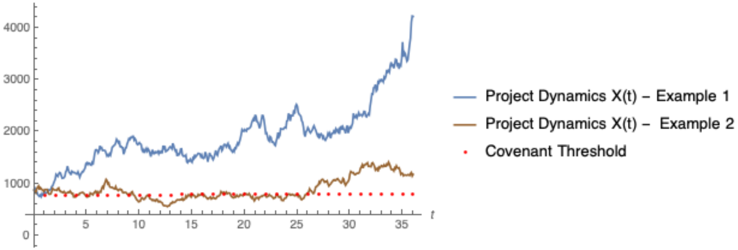

Figure: Examples of possible dynamics of the project value together with the restrictive covenant boundary.

Decision Tree

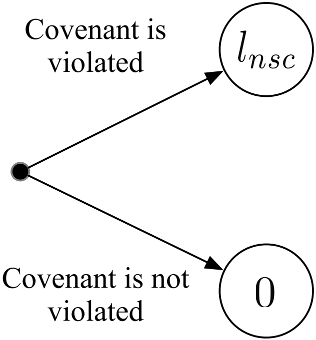

Figure: Decision tree in each point in time

Continuous Time Model

- Expected losses for a bank

$$el_{nsc}(t) = PCV(t) l_{nsc}(t) = PCV(t) \left(S_T - X_t e^{r_S (T-t)} \right)$$

- Probability of covenant violation

$$PCV(t) = P(X_t < \alpha(t)) = \Phi \left(\frac{ln \frac{\alpha(T)}{X(t)} - (\mu - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \right) +$$

$$+ \left(\frac{X(t)}{\alpha(t)} \right)^{1-2\frac{\mu - r}{\sigma^2}} \Phi \left(\frac{ln \frac{\alpha^2(t)}{X(t)\alpha(T)} - (\mu - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \right)$$

Expected losses for a bank

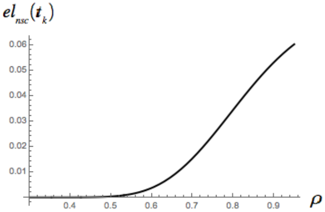

Figure: The dependency of the expected losses when non selling the collateral on covenant strength

Cost Function

$$c(\rho) = c_0 e^{-c_1 \rho}$$

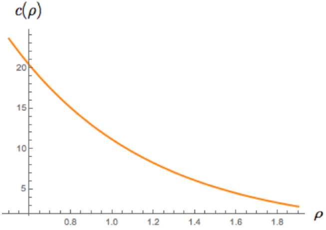

Figure: Cost function

Source: S.Das and S.Kim. Credit spreads with dynamic debt. Journal of Banking and Finance, 50(C):121–140, January 2015.

Optimal Covenant Strength

$$PCV(t)\left(S_T - X_t e^{r_S(T-t)}\right) = c_0 e^{-c_1\rho} \tag{1}$$

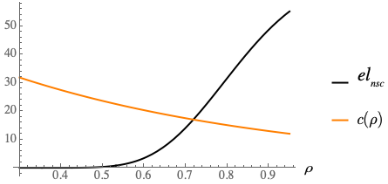

Figure: The optimal level of covenant strength as a tradeoff between increasing the expected losses and decreasing the costs

Sensitivity Analysis - Drift

By decreasing the drift value μ from 1.56% to 0.20%, the optimal covenant strength index becomes less strict (decreases from 0.718 to 0.703)

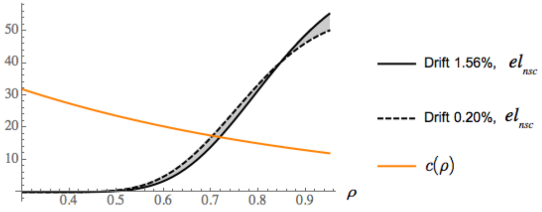

Figure: The change in optimal level of covenant strength with respect to the drift value

Sensitivity Analysis - Volatility

If we consider more volatile situation in the market (increase of σ from 11.25% to 15.00%), the level of optimal covenant strength decreases from 0.718 to 0.681

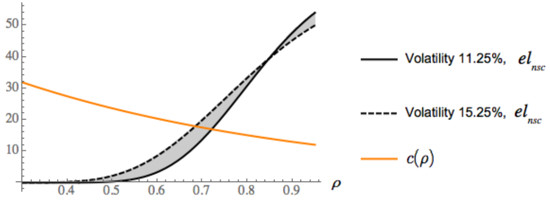

Figure: The change in optimal level of covenant strength with respect to the volatility value

Information Asymmetry

Kaplan and Strömberg (2004) classification of agency problems

1. The effort of the entrepreneur is unobservable (the moral hazard approach predicts the dependence of the entrepreneur's compensation on performance)
2. Asymmetry in the knowledge about the entrepreneur's quality (the solution is also greater pay-for-performance)
3. The pursuit of the decision right in case of disagreement with the entrepreneur after the investment (control theories suggest that the conclusion is state dependent)
4. "Hold-up" problem, when the entrepreneur threatens to leave the venture in case the human capital is very important (the solution of this problem is in vesting the shares to the entrepreneur)

Information Asymmetry — Motivation

- Gertler (1992): in IA setting covenant strength increases in bad states of the world
- Roberts (2015): when IA increases, renegotiations occur more frequently
- Dessein (2005): although formal control increases when IA increases, real control decreases
- Gârleanu and Zwiebel (2009):
    - fixed probability p of good state of the world - more control goes to the lender
    - p goes down — more control goes to the entrepreneur
    - p goes up - more control goes to the entrepreneur when information asymmetry increases and more control goes to the lender when costs increase

Information Asymmetry — Decision Tree

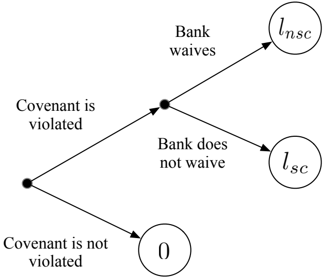

Figure: Decision tree in each point in time

Two types of Information Asymmetry

1. Behaviour of a project

$$el_{nsc}(t) = PCV(t)l_{nsc}(t) = PCV(t)\left(S_T - X_t e^{r_S(T-t)}\right)$$

$$el_{sc}(t) = PCV(t)l_{sc}(t) = PCV(t)\left(S_T - X_t e^{r(T-t)}\right)$$

Total expected losses depend on

$$\lambda_t = \frac{X_t}{\alpha_t}$$

$$el_{ai}(t) = \left(1 - \frac{1}{\lambda_t/\sigma}\right) el_{sc}(t) + \frac{1}{\lambda_t/\sigma} el_{nsc}(t),$$

Two types of Information Asymmetry

2. Behaviour of a manager

- two different volatility variables in the model: σ_f - is project-specific and σ_b - based on the behaviour of the manager

$$el_{ai,bf}(t) = \left(1 - \frac{1}{\lambda_t/\sigma_b}\right) el_{sc,f}(t) + \frac{1}{\lambda_t/\sigma_b} el_{nsc,f}(t). \quad (2)$$

Optimal Covenant Strength under IA

Compared to the baseline model, covenant strictness index in the model with information asymmetry decreases from 0.718 to 0.701

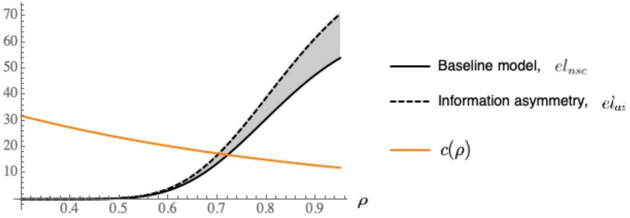

Figure: Information asymmetry - behaviour of the project

Information Asymmetry Type I — Big Volatility

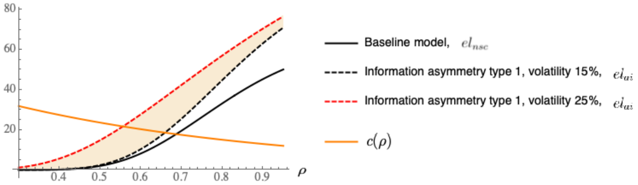

Figure: Information asymmetry - behaviour of the project - volatility increases from 15% to 25%

Information Asymmetry Type I — Small Volatility

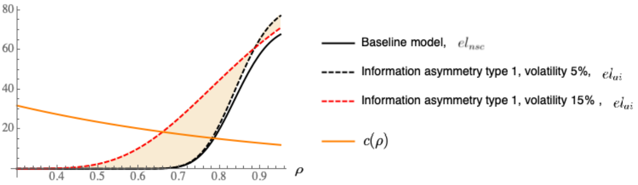

Figure: Information asymmetry - behaviour of the project - volatility increases from 5% to 15%

Information Asymmetry Type II — Big Volatility

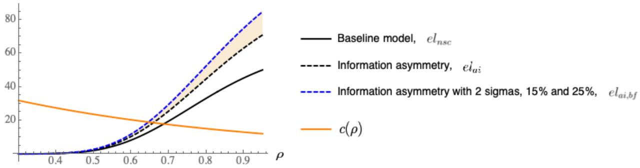

Figure: The comparison of the 2 cases of information asymmetry

Information Asymmetry Type II — Small Volatility

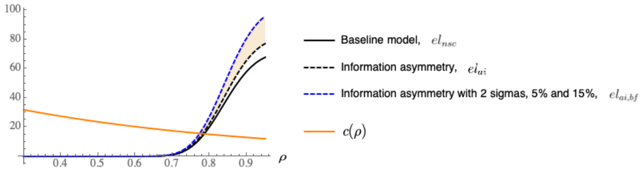

Figure: The comparison of the 2 cases of information asymmetry

Simulations

1. A framework for measuring various risk-parameters of a project (average number of covenant violations per contract, frequency of covenant violation, frequency of loan repayment)
2. Implementation of different decision rules for a bank and based on the risk-parameters assessing the effect of those decisions
3. A recursive technique for determining the level of covenant strength that allows the bank to maintain the performance of a specific risk-parameter

Parameters Estimation

- **Step 1.** $n-1$ rollover dates, $t_1, \ldots, t_{n-1}$, at which the bank checks whether the covenant is violated
- **Step 2.** Define the covenant threshold for each of these dates t_i
- **Step 3.** Do Monte Carlo simulation of the project process for N times
- **Step 4.** At each rollover date check whether the covenant is violated. If the covenant is violated, set the value of the corresponding element in the matrix of indicators as 1, otherwise as 0

Parameters Estimation

- **Step 5.** Estimate covenant violation frequency f_{CV}

$$f_{CV} = \frac{n_{CV}}{N} \tag{3}$$

n_{CV} — the number of contracts with at least one covenant violation

- **Step 6.** Estimate the average number of covenant violations per contract ν_{CV}.

Simulation Results — Baseline Simulation

Parameter	Baseline Simulation
The average number of covenant violations per contract ν_{CV}	4.4
Frequency of covenant violation f_{CV}	0.39
Number of contracts with at least one covenant violation $n_{CV}(N = 100\ 000)$	38 642

Figure: The Results of Baseline Simulation Model

Simulation Results — Interval Simulation

Parameter	Baseline Simulation	Interval Simulation
The average number of covenant violations per contract ν_{CV}	4.4	0.7
Frequency of covenant violation f_{CV}	0.39	0.39
Number of contracts with at least one covenant violation n_{CV}	38 616	38 616

Figure: The Results of Baseline Simulation Model and Interval Simulation Model

Simulation Results — Repayment Frequency

Parameter	Baseline Simulation	Interval Simulation
The average number of covenant violations per contract ν_{CV}	4.4	0.7
Frequency of covenant violation f_{CV}	0.39	0.39
Number of contracts with at least one covenant violation n_{CV}	38 616	38 616
Frequency of loan repayment f_{LR}	0.83	0.83

Figure: The Results of Baseline Simulation Model and Interval Simulation Model

Sensitivity Analysis — Drift

Parameter	$\mu = 0.0056$	$\mu = 0.0106$	$\mu = 0.0156$	$\mu = 0.0206$
Baseline Simulation				
ν_{CV}	7.9	5.9	4.4	3.1
f_{CV}	0.57	0.48	0.39	0.30
n_{CV}	56 684	47 506	38 616	30 366
f_{LR}	0.66	0.75	0.83	0.89
Interval Simulation				
ν_{CV}^{int}	1.0	0.9	0.7	0.6
f_{CV}^{int}	0.57	0.48	0.39	0.30
n_{CV}^{int}	56 684	47 506	38 616	30 366
f_{LR}^{int}	0.66	0.75	0.83	0.89

Figure: The Dynamics with Respect to the Drift

Sensitivity Analysis — Volatility

Parameter	$\sigma = 0.0525$	$\sigma = 0.0825$	$\sigma = 0.1125$	$\sigma = 0.1525$
Baseline Simulation				
ν_{CV}	0.10	1.6	4.4	8.3
f_{CV}	0.02	0.18	0.39	0.59
n_{CV}	1 708	17 925	38 616	59 145
f_{LR}	0.99	0.94	0.83	0.69
Interval Simulation				
ν_{CV}^{int}	1.0	0.9	0.7	0.6
f_{CV}^{int}	0.02	0.18	0.39	0.59
n_{CV}^{int}	1 708	17 925	38 616	59 145
f_{LR}^{int}	0.99	0.94	0.83	0.69

Figure: The Dynamics with Respect to the Volatility

Recursive Technique

- **Step 1.** Determine all necessary variables.
- **Step 2.** Run the simulation of the process dynamics as Brownian motion for N times.
- **Step 3.** Determine the covenant threshold. Determine on which rollover dates the covenant threshold is maintained. Calculate the relative number of non violations at maturity and check whether this ratio is greater or equal to a certain pre-specified level. If yes — stop the cycle, if not — reduce the covenant index by 0.05 and repeat Step 3 until the relative number of covenant non violations at maturity meets the requirement

Assume the bank wants to maintain the probability of repayment of 85% level. As a result, we obtain the covenant strength index equal to 0.65.

Renegotiation of the Covenant Strength

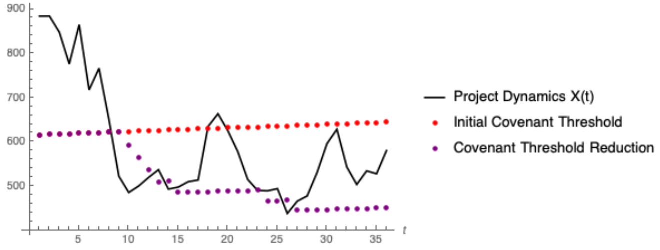

Figure: The simulation model with covenant threshold reduction in case of covenant violation

Compared to the average number of violations per contract in a baseline simulation setting (4.4), with possibility of covenant reduction this estimation drops down to 2.2.

Conclusions

- A theoretical framework that allows to determine the optimal covenant strength index, both under symmetric and under asymmetric information
- A framework for accessing the consequences of covenant violation in Monte Carlo simulation